تم الرفع بواسطة
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Calculus 2
Second

[redacted] Palestine Technical University
Department of Applied Mathematics.
Calculus II.

Student # معن سلام محمود قعدان **Second Exam** May, 19^{th}, 2010.

Student name بالعربيه: Time: 60 min.

احد/ثلاثاء/خميس 12:00 - 1:00

Instructors: Dr. Khalida Nazzal. Mrs. Kifaya Ayash.
Mr. Jasem Badran. Mr. Saleh abou Serees.

Problem 1. (40 pts.) Circle the correct answer.

1) If $\cosh x + \sinh x = 1$, *then* $x =$

a) 3
b) 2
c) 1
d) 0.

$\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = 1$

$\frac{2e^x}{2} = 1$

2) $\int \frac{2x}{2x+5} dx =$

a) $x - \ln|2x+5| + C$.
b) $\ln|2x+5| + C$.
c) $x + \frac{5}{2}\ln|2x+5| + C$
d) $x - \frac{5}{2}\ln|2x+5| + C$

3) The Series $\sum_{0}^{\infty} \frac{1}{(n+1)(n+2)}$

a) Converges to 1
b) Converges to 0
c) Converges to $\frac{1}{2}$
d) Diverges

$\frac{A}{n+1} + \frac{B}{n+2}$

$A(n+2) + B(n+1) = 1$

$A = 1$, $B = -1$

4) $\sinh(5\ln x) =$

a) $\frac{5}{2}(x - \frac{1}{x})$
b) $\frac{1}{2}(x^5 - \frac{1}{x^5})$
c) $\frac{1}{2}(x^5 + \frac{1}{x^5})$
d) $2x$.

$\frac{e^x - e^{-x}}{2} = \frac{e^{5\ln x} - e^{-5\ln x}}{2} = \frac{e^{\ln x^5} - e^{\ln x^{-5}}}{2}$

$\frac{x^5 - \frac{1}{x^5}}{2} = \frac{1}{2}(x^5 - \frac{1}{x^5})$

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5) Consider $\sum_{1}^{\infty} a_n$ Where $a_n \geq 0$ Then

a) If $\lim_{x \to \infty} a_n = 0$ then $\sum_{1}^{\infty} a_n$ converges.

b) If $\lim_{x \to \infty} a_n \neq 0$ then $\sum_{1}^{\infty} a_n$ diverges

c) If $\sum_{1}^{\infty} a_n$ diverges then $\lim_{x \to \infty} a_n \neq 0$

d) If $\sum_{1}^{\infty} a_n$ converges then $\lim_{x \to \infty} a_n \neq 0$

6) The sequence $(a_n) = (1 - \frac{1}{n^2})^n$

a) Converges to 1

b) Converges to e

c) Converges to e^{-1}

d) diverges

7) $\int \tan^{-1} x \, dx =$

a) $\ln(\cos^{-1} x) + c$

b) $x \tan^{-1} x - \ln(x^2 + 1) + c$

c) $x \tan^{-1} x - \ln \sqrt{x^2 + 1} + c$

d) $x \tan^{-1} x - \dfrac{x}{x^2 + 1} + c$

8) $\sum_{1}^{\infty} (\ln(x))^n$ Converges If

a) $-1 < x < 1$

b) $0 < x < e$

c) $0 < x < 1$

d) $e^{-1} < x < e$

9) $\int_{\ln 2}^{\ln 7} 6e^t \cosh t \, dt =$

a) $\frac{135}{2}$

(b) $\frac{135}{2} + 3\ln 7 - 3\ln 2$

c) $3\ln\frac{7}{3}$

d) 1.

$e^{2t} = 2e^{2t}$

$\int 3e^t\left(\frac{e^t + e^{-t}}{2}\right)$

$\int 3e^{2t} + 3(1)$

$\int 3e^{2t} + 3 \quad \int 3(e^{2t}+1)$

$3\int e^{2t}+1 = 3\left[\frac{e^{2t}}{2} + t\right]_{\ln 2}^{\ln 7}$

$3\left[\frac{e^{\ln 7^2}}{2} + \ln 7 - \left(\frac{e^{\ln 2^2}}{2} + \ln 2\right]\right.$

$3\left[\frac{49}{2} + \ln 7 - \frac{4}{2} + \ln 2\right]$

$3\left[\frac{45}{2} + \ln 7 - \ln 2\right]$

$\boxed{\frac{135}{2} + 3\ln 7 - 3\ln 2}$

10) The Series $\sum_{1}^{\infty} \frac{2^n - 2}{3^n}$

(a) Converges to 1

b) Converges to $\frac{3}{4}$

c) Converges to 2

d) Converges to $\frac{3}{2}$

$\sum \frac{2^n}{3^n} - \sum \frac{2}{3^n}$

$\sum_1^\infty \left(\frac{2}{3}\right)^n - 2\sum_1 \left(\frac{1}{3}\right)^n$

$\frac{2}{3} + \frac{4}{9} \quad 2\left(\frac{1}{3}\right)^1 + 2\left(\frac{1}{9}\right)$

$\frac{a}{1-r} \quad \frac{\frac{2}{3}}{1-\frac{2}{3}} - \frac{\frac{2}{3}}{1-\frac{1}{3}}$

$2 - 1 = 1$

Problem 2. (30 pts) Compute

a) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

$a^2 - x^2 \Rightarrow (2)^2 - x^2$

$\Rightarrow \boxed{x = 2\sin\theta} \Rightarrow \sin\theta = \frac{x}{2}$

$dx = 2\cos\theta \, d\theta$

$\boxed{1 - \sin^2\theta = \cos^2\theta}$

$\int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{\sqrt{4 - 4\sin^2\theta}} = \int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{\sqrt{4(1-\sin^2 x)}}$

$= \int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{\sqrt{4\cos^2\theta}} = \int \frac{4\sin^2\theta\,[2\cos\theta\, d\theta]}{2\cos\theta}$

$= \int 4\sin^2\theta \Rightarrow \boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$

$= 4\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta = 2\int 1 - \cos 2\theta \, d\theta$

$= 2\left[\theta - \frac{\sin 2\theta}{2} + c\right] = \boxed{2\theta - \sin 2\theta + C}$

$\theta = \sin^{-1}(x) \quad \sin 2\theta = x \Rightarrow \boxed{I = 2\sin^{-1}(x) - x + C}$

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b) $\int \sin^3 x \cos^2 x \, dx$ $= \int \sin^2 x \cos^2 x \cdot \sin x$

$= \int (1-\cos^2 x)(\cos^2 x) \cdot \sin x$

$u = \cos x \Rightarrow du = -\sin x \, dx$

$I = -\int (1-u^2)\, u^2 \, du = -\int u^2 - u^4 \, du$

$= -\left[\frac{u^3}{3} - \frac{u^5}{5}\right] = \frac{-u^3}{3} + \frac{u^5}{5} + c = \boxed{-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C}$

c) $\int \frac{y^2+2y+1}{(y^2+1)^2} dy.$

~~$\int \frac{(y+1)^2}{y^2}$~~ $= \frac{Ay+B}{(y^2+1)} + \frac{Cy+D}{(y^2+1)^2}$

$y^2+2y+1 = Ay+B + (Cy+D)(y^2+1)$

$y^2+2y+1 = Ay+B+[Cy^3+Cy+Dy^2+D]$

$y^2+2y+1 = (A+C)y + Dy^2 + Cy^3 + (B+D)$

$(A+C)y = 2y \Rightarrow \boxed{A+C=2}$ $\quad Cy^3 = 0 \Rightarrow C=0$

$Dy^2 = y^2 \Rightarrow \boxed{D=1}$ $\quad (B+D)=1$

$B+1=1 \Rightarrow \boxed{B=0}$

$A+C=2 \Rightarrow A+0=2 \Rightarrow \boxed{A=2}$

$\boxed{A=2}$ $\boxed{B=0}$ $\boxed{C=0}$ $\boxed{D=1}$

$I: \int \frac{2y}{y^2+1} + \frac{1}{(y^2+1)^2} = \boxed{\ln|y^2+1| + \int \frac{1}{(y^2+1)^2} + C}$

~~$=\int (y^2+1)^{-2}$~~ ~~$u = y^2 \Rightarrow du = 2y$~~

$=$

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هذا السؤال
(ضيقة الوقت)

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Problem 3. (30 pts) Determine which of the following integral (or series) converges and which diverges.

a) $\int_{-1}^{1} \frac{dx}{x^{\frac{2}{3}}}$.

$= \lim_{n\to\infty} \int_n^1 x^{-\frac{2}{3}} = \lim_{n\to\infty} \int_n^1 \frac{x^{\frac{1}{3}}}{\frac{1}{3}}$

$= 3 \lim_{n\to\infty} \sqrt[3]{x} = 3(\infty) = \infty$

diverge

b) $\int_{1}^{\infty} \frac{dx}{\sqrt{e^x - x}}$

$\Rightarrow \frac{1}{\sqrt{e^x}}$ Compare

$\frac{\sqrt{e^x}}{\sqrt{e^x - x}} = \sqrt{\frac{e^x}{e^x - x}} = \sqrt{1} = 1$

How

Converge

c) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^{100}}$

$\frac{1}{n \ln n}$

$\frac{1}{n}$ Compare

$n > \ln n$ $n > 1$

diverge

P-series $P = 1$